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## Effects of Differences in Mineralization on the Mechanical Properties of Bone

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## Effects of differences in mineralization on the mechanical properties of bone

BY J. D. CURREY

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[Plate 1]

There is a considerable variation in the mineralization of bone; normal, non-pathological compact bone has ash masses ranging from 45 to 85% by mass. This range of mineralization results in an even greater range of mechanical properties. The Young modulus of elasticity can range from 4 to 32 GPa, bending strength from 50 to 300 MPa, and the work of fracture from 200 to 7000 Jm<sup>-2</sup>. It is not possible for any one type of bone to have high values for all three properties. Very high values of mineralization produce high values of Young modulus but low values of work of fracture (which is a measure of fracture toughness). Rather low values of mineralization are associated with high values of work of fracture but low values of Young modulus and intermediate values of bending strength.

The reason for the high value for the Young modulus associated with high mineralization is intuitively obvious, but has not yet been rigorously modelled. The low fracture toughness associated with high mineralization may be caused by the failure of various crack-stopping mechanisms that can act when the mineral crystals in bone have not coalesced, but which become ineffective when the volume fraction of mineral becomes too high.

The adoption of different degrees of mineralization by different bones, leading to different sets of mechanical properties, is shown to be adaptive in most cases studied, but some puzzles still remain.

### INTRODUCTION

The papers by Dr Miller and Dr Glimcher in this symposium have given an account of the main organic and mineral phases of bone. In this paper I shall show how the relative amounts of the two phases can profoundly modify the mechanical properties of bone.

It is intuitively obvious that bone with a large volume fraction of pores will have a lower modulus of elasticity, and be weaker, than bone without significant cavities. This is indeed so; cancellous bone has been shown by Carter and his colleagues to have a stiffness proportional to the cube of the volume fraction of bone material (as opposed to marrow) and tensile and compressive strengths proportional to the square of the volume fraction (Carter & Hayes 1976; Carter *et al.* 1980). However, probably of more interest in the present context, and perhaps more surprising, is the great effect that differences in the mineral content of bone have on its mechanical properties.

### THE YOUNG MODULUS OF ELASTICITY

Figure 1 shows the relationship between the Young modulus of elasticity and the mineral content of compact bone tested wet. The mineral content was measured merely by finding the mass remaining after the bone had been ashed in a platinum boat. The individual points are

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not shown, just the envelopes enclosing the points. Two things are at once apparent from this diagram. One is that bone shows an enormous range of values of Young modulus, in this data set from about 4 GPa to 33 GPa. The other is that although the Young modulus increases monotonically with mineral content, they are not proportional to each other. The bone with the lowest Young modulus, some bone from red deer's antler, has over 45% mineral by mass, equivalent to a mineral volume fraction of about 20%, yet has a modulus of only 4 GPa.

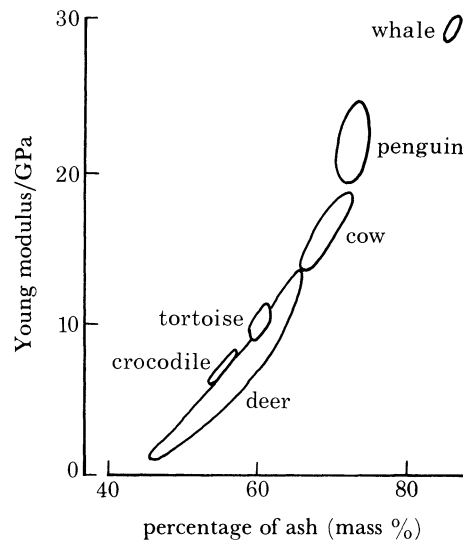


FIGURE 1. Relation between the Young modulus of elasticity (measured in bending) and the mineral content of bone determined from the mass remaining after ashing. The samples were all cortical bone specimens. Only the envelopes enclosing the points are shown.

This nearly tenfold range of values of Young modulus in a material that histologically does not show a great deal of variation is remarkable, and needs to be explained, both from the point of view of the mechanism producing it, and the adaptive reason for it.

#### MODELLING THE YOUNG MODULUS

The Young modulus of tendon, which we can take to represent collagen in a fairly uniformly aligned arrangement, is about 2 GPa. The Young modulus of hydroxyapatite is about 110 GPa. The problem is to obtain a model that will explain the elastic behaviour of bone by using this information, and other information, such as the volume fraction of mineral, the anisotropy of the elastic properties, and the detailed arrangement of the microscopic constituents of bone.

Katz (1971, 1981) and his co-workers have been particularly concerned with this, but are still far from success. Katz started by modelling bone as if the mineral and the collagen formed a multilayer sandwich. If a sandwich is loaded parallel to the layers the *stiffnesses* of the two components are additive; this is the so-called Voigt model. If the sandwich is loaded normal to the layers the *compliances* are additive. This is the so-called Reuss model. If bone is considered to be a multilayered composite material, made of layers of collagen and layers of mineral of modulus  $E_c$  and  $E_m$ , and of relative volumes  $V_c$  and  $V_m$ , respectively, then for the Voigt model,

loading parallel to the layers, the modulus of the bone

$$E_b = E_c V_c + E_m E_m,$$

and for the Reuss model:

$$1/E_b = V_c/E_c + V_m/E_m.$$

Katz found that bone and enamel do fit within the upper and the lower bounds of this model, but the bounds themselves were so far apart that bone could hardly be said to be satisfactorily modelled (Katz 1971). Later, Katz refined his model by assuming (which is almost certainly true) that the apatite crystals are oriented along the collagen fibrils and tightly bonded to them. The collagen fibrils will, however, themselves be at various orientations to the direction of loading. Following Krenchel (1964), Katz developed the equation

$$E_b = E_c V_c (1 - \nu_c \nu_b) / (1 - \nu_c^2) + \sum E_m V_m \alpha_n (\cos^4 \phi_n - \nu_b \cos^2 \phi_n \sin^2 \phi_n).$$

Subscript c refers to collagen, m to mineral,  $\nu$  is the Poisson ratio,  $\alpha_n$  is the fraction of mineral crystallites that lie at an angle  $\phi_n$  from the direction of loading. The first part of the equation refers to the collagen, which is assumed to have the same modulus in all directions. This, though not true, does not matter because the Young modulus of collagen is so low. The second part of the equation gives the effect of the mineral.

This equation is not really satisfactory for two reasons. One is that, as Katz (1981) shows, the model gives too low a value for the Young modulus when the fibres are loaded at a large angle to their length. When this happens  $\cos^4 \phi_n$  approaches zero, and the bone should have a Young modulus near that of collagen. In fact the anisotropy of the Young modulus of bone rarely seems to exceed a factor of two. The other problem with this equation is that, because the contribution of the collagen to the stiffness is so small, the Young modulus should be roughly proportional to the volume fraction of the mineral. But in antler with a volume fraction of about 30% the Young modulus is about 7 GPa, while in the penguin's humerus, with a mineral volume fraction of about 50%, the Young modulus is about 28 GPa. It is possible to get round this difficulty by assuming that the crystalline direction in antler is more misaligned than in the penguin's bone. This might be true, but certainly cannot be true for the whale tympanic bulla, which appears nearly isotropic when examined with polarized light; yet this bone has a very high Young modulus. Furthermore, because the model has difficulty in predicting anisotropy, these same predictions of anisotropy should not be invoked to explain the difference between antler and other bone.

Katz assumes that the apatite needles or plates are bound rigidly to the collagen, which is reasonable, but makes no allowance for their length; the equations he uses are for very high aspect ratio fibres. Incorporating the length of fibres in models of composite materials is difficult, and the problem has not been solved yet except for very simple cases, and bone is certainly not simple.

The analyses of Katz and his co-workers have all assumed values for the Young modulus of collagen that are the same as those of collagen tested on its own. However, there have been objections to this. McCutchen (1975) proposed that the mineral phase of bone 'straitjackets' the collagen and prevents it from lengthening under stress. Hukins (1978) proposed a modification of McCutchen's model in which the collagen fibrils are considered to be arranged as liquid crystals whose habitual reorientation under stress is prevented by the mineral. These suggestions are ingenious, but until they can be quantified it is probably best to consider them

just as suggestions to be borne in mind when considering more quantified models. Certainly there is such an intimate relation between the collagen and the mineral at the nanometre level that it is doubtful to what extent we should consider collagen having any of the mechanical properties we know of from tests of it in isolation.

#### STRENGTH AND TOUGHNESS

Differences in mineralization clearly have a profound effect on the Young modulus of elasticity of bone. They also have effects on the fracture properties, and it is not so obvious why this should be so. Figure 2 shows the relation between the mineral content of bone

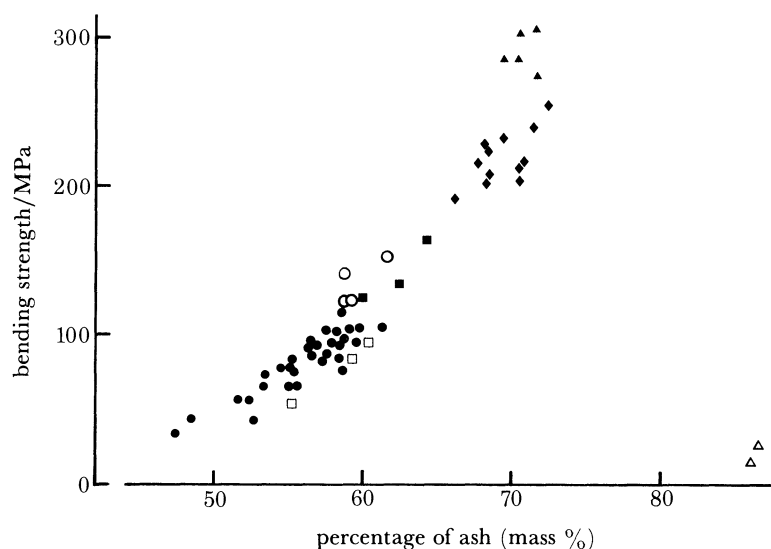


FIGURE 2. Relation between bending strength and the mineral content of bone specimens: ● red deer antler; □ crocodile skull bone; ○ Galapagos tortoise femur; ■ Muntjac antler; ◆ cow femur; ▲ King penguin humerus; △ tympanic bulla of fin whale.

specimens of various bones, and their bending strength. Over much of the range there is a monotonically increasing relation, as there is between Young modulus and mineral content. However, this relation breaks down with the bone with the greatest amount of mineralization (the tympanic bulla of the whale), which has a very low bending strength as well as having the highest mineral content of the bones tested. Figure 3 shows that the relation between bending strength and Young modulus is almost one of proportionality except that, again, the whale tympanic bulla lies off the main scatter.

Bending strength tests, though convenient to perform, and mimicking loading situations occurring in life, are analytically rather untidy, because two properties of the specimens are being confounded. These are the stress at which the material yields in tension, and also the amount of post-yield deformation the material can undergo before it breaks in two. An increase in either of these will increase the bending moment at failure, which is the experimental variable used to calculate bending strength. It is not possible easily to separate these two quite different properties. However, it does appear, from inspection of the load–deformation curves of these specimens, that specimens with very high bending strengths yielded at high stress, but did not

undergo much post-yield deformation. Specimens with low bending strengths yielded at a low stress, and underwent considerable post-yield deformation. Specimens of intermediate strength yielded at intermediate stress, but also underwent considerable post-yield deformation.

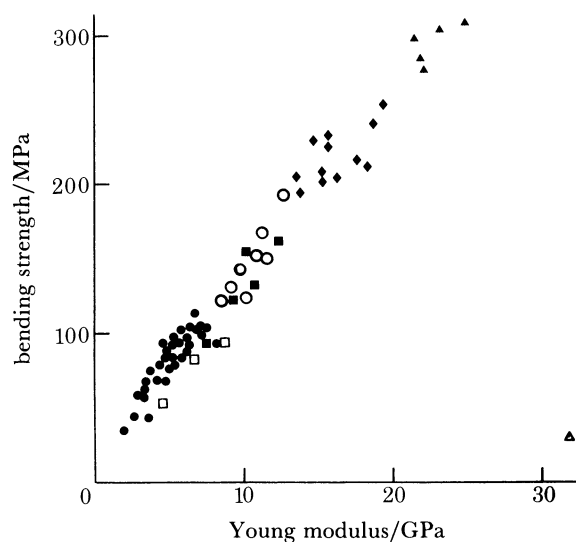


FIGURE 3. Relation between bending strength of cortical bone specimens and the Young modulus of the same specimens: ● red deer antler; □ crocodile skull bone; ○ Galapagos tortoise femur; ■ Muntjac antler; ◆ cow femur; ▲ King penguin humerus; △ tympanic bulla of fin whale.

Bending strength tests, and tensile tests, are performed by slowly loading carefully prepared specimens that do not have significant flaws and notches on their surfaces. Two factors make such tests unreliable guides to how bone will behave in life. One is that when bones break they usually do so not because they have been loaded by too great a *force*, but because they have had to absorb too much kinetic *energy* resulting from a fall or a blow. Both the maximum load borne, which is measured in a bending test, *and* the total deformation of the bone (or bone specimen) will increase the energy absorbed. The second factor is that in real life bones are not entirely smooth, and will have small flaws in them. These flaws tend to be sites where a crack can start, and perhaps travel through the specimen. The whole field of 'fracture mechanics' aims to determine the circumstances in which a crack may or may not spread under load. The ability of a material to prevent cracks travelling through itself is called its 'fracture toughness', and is a good measure of strength in the rough-and-tumble of the real world.

Figure 4 shows how different types of bone perform in respect of the Young modulus, bending strength, and fracture toughness. It seems that it is not possible for bone to perform well in all three properties. In particular, once a reasonably large Young modulus has been achieved, there is an inverse relation between Young modulus and fracture toughness.

Work of fracture experiments are done in a way that produces low and controllable velocities of crack travel. In life, cracks travel very fast and uncontrollably. Nevertheless there is a strong relation between fracture toughness and the impact resisting ability of plain, smooth, specimens. For instance, at the extremes, it is very difficult to break machined specimens of red deer's antler in impact, while the performance of whale's tympanic bulla in impact is pathetic.

Because increase in Young modulus goes with increase in mineral content, it is reasonable

to suppose that high mineralization may be a cause of low fracture toughness. There is some evidence that this is so. High fracture toughness depends on the ability of a material to make crack travel an energy-expensive process. A crack travelling through a brittle material, such as glass, has to provide little more energy than the surface energy of the two new surfaces formed. These surfaces are smooth, and the work of fracture is in the region of  $10 \text{ Jm}^{-2}$ , orders of magnitude less than that of bone.

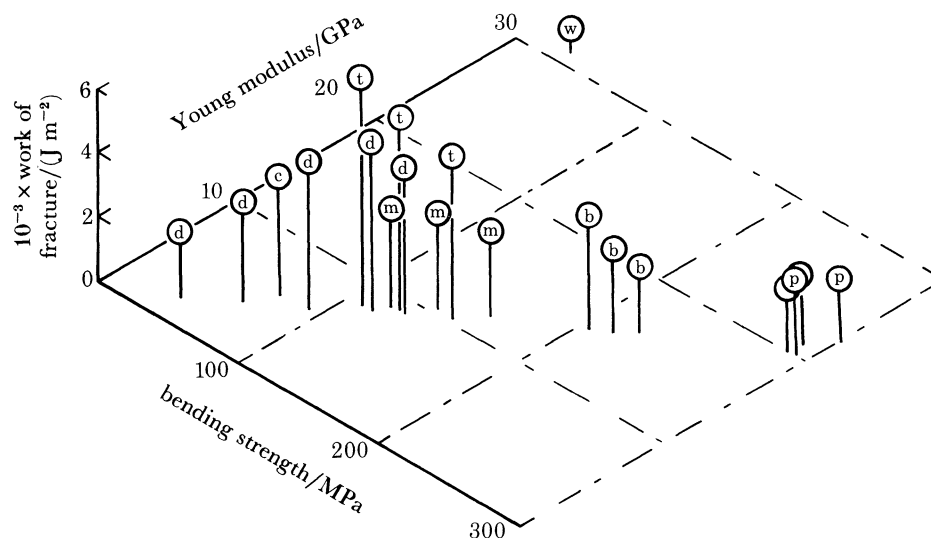


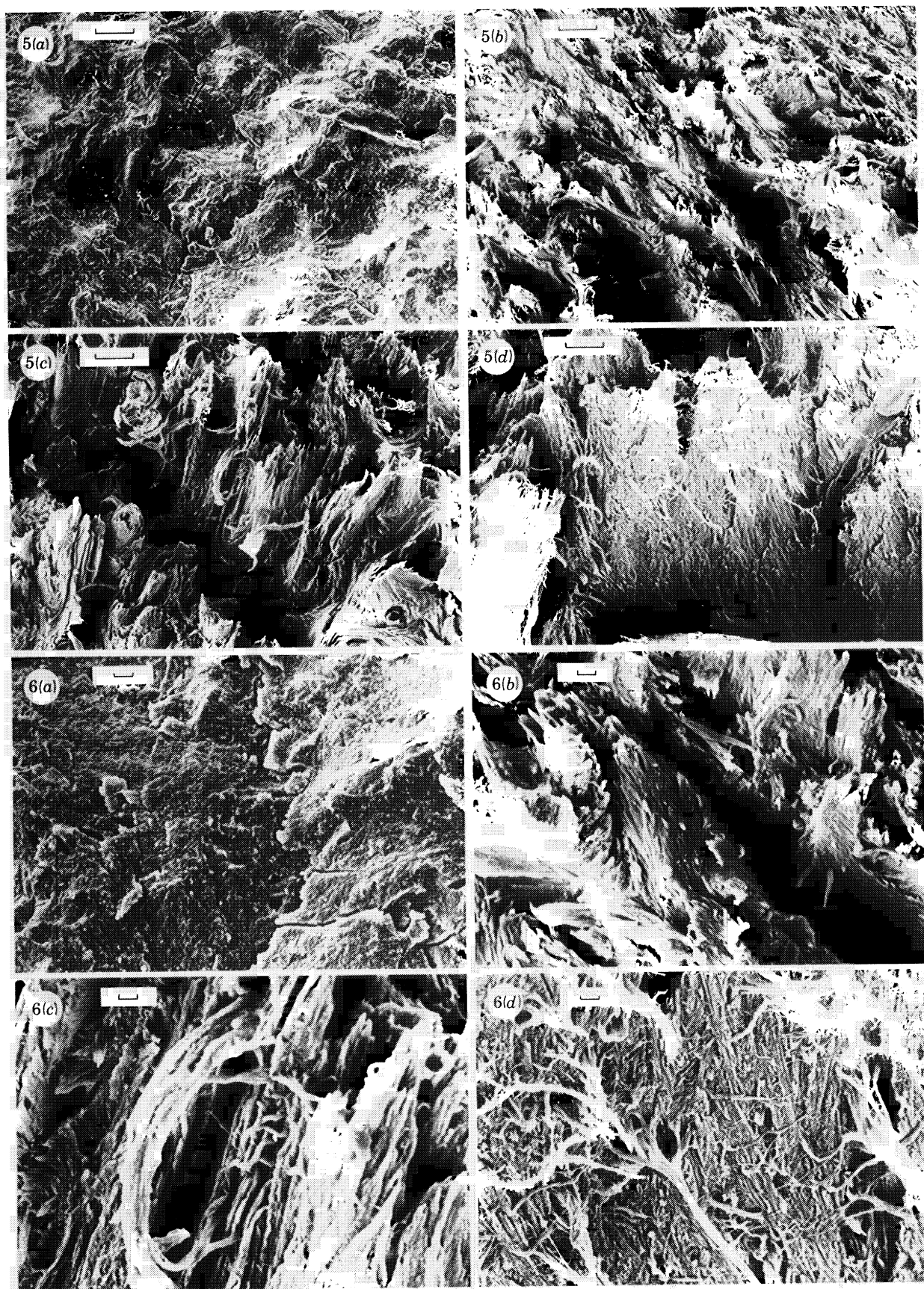
FIGURE 4. Relation between three mechanical properties. Each circle refers to the values from two specimens taken from adjacent sites in the bone. (It is not possible to obtain values of work of fracture and bending strength from the same specimen.) The work of fracture was obtained from the static loading of notched specimens, by using the method of Tattersall & Tappin (1966). The letters in the circles refer to the following species: d, red deer antler; c, crocodile skull bone; t, Galapagos tortoise femur; m, Muntjac antler; b, femur of cow; p, King penguin; w, tympanic bulla of fin whale.

Crack travel becomes energetically expensive if the fracture surface is rough, because this greatly increases the actual new surface area that must be created relative to the nominal cross sectional area of the specimen. If the fracture surface is fibrous, this will show that energy will have been used in tearing the fibres from their surrounding matrix. As well as these surface appearances, a very rough surface is an indication that small-scale fissuring will have occurred underneath the surface and these fissures, too, will absorb energy. Figures 5 and 6 (Plate 1) show fracture surfaces of four different types of bone, and show the obvious relation between mineral content, energy absorption, and the appearance of the surface.

#### DESCRIPTION OF PLATE 1

FIGURE 5. Scanning electron micrographs of fracture surfaces of bone specimens tested for work of fracture. The scale bar in each represents  $100 \mu\text{m}$ . (a) Whale tympanic bulla; work of fracture  $210 \text{ Jm}^{-2}$ ; ash content 86.4%. (b) Cow femur; work of fracture  $2630 \text{ Jm}^{-2}$ ; ash content 66.6%. (c) Red deer antler; work of fracture  $4680 \text{ Jm}^{-2}$ ; ash content 57.5%. (d) Galapagos tortoise femur; work of fracture  $6100 \text{ Jm}^{-2}$ ; ash content 58.3%. All percentages by mass.

FIGURE 6. The same specimens as shown in plate 1, but at a larger scale. The scale bar in each represents  $10 \mu\text{m}$ .



FIGURES 5 and 6. For description see opposite.

(Facing p. 514)



## MODELLING STRENGTH AND TOUGHNESS

Some attempts have been made to explain the fracture behaviour of bone by comparing it with ordinary composite materials. In doing this we meet the problem of the level at which the comparison should be made. At one level we could consider the collagen to be the matrix, in which the mineral 'fibres' are embedded. At the next level we could consider the collagen fibrils, with their associated mineral, to be the fibres, each more or less loosely connected to neighbouring fibres. At the next level we could consider the bony lamellae to be the 'fibres', being separated as they are by interlamellar regions, which are relatively collagen-poor, having a high proportion of ground substance of dubious chemical constitution. At the highest level we could consider the secondary osteones or laminae, characteristic of adult bone, to be the fibres, again more or less loosely connected to each other by cement lines and other structures.

Secondary osteones, or Haversian systems, are structures formed by the internal remodelling of bone. A cylindrical channel about 200  $\mu\text{m}$  across is formed round a blood vessel and then filled with concentric lamellae until the blood vessel is tightly enclosed again. The resulting structure resembles a miniature leak *Allium porrum*. In some species, particularly man, bone is continually being remodelled in this way, so that the bony tissue is nothing but a mass of complete Haversian systems, and the incomplete remnants of previous generations of systems (de Riquès 1979).

The early analogies between composite materials and bone (Currey 1964; Mack 1964) concentrated on the lowest level, that of collagen as the matrix and the mineral as the fibre. This turns out not to have been very fruitful. The reason is that at such a low level, with such an intimate relation between the mineral and the protein, it is impossible to have much idea of how the collagen is behaving. In the discussion of elasticity I mentioned the possibility that the properties of collagen in bone may be radically different from those that it has in, say, tendon (Lees & Davidson 1977; Hukins 1978; McCutchen 1975). At the moment we can say little about the aspect ratio and size of the mineral, because we know little about the mineral morphology, and little about the mode of attachment of the collagen to the mineral. Such information is necessary to model materials by composite theory. Furthermore, at this level we cannot see the fracture surface clearly enough to make out, for instance, whether the minute mineral blocks pull out from the matrix. It seems to me likely that the composite nature of bone is important, at this level, for fracture as well as for elasticity, but that we shall have to wait a good while before we know much.

At the next higher level things are better because the individual fibres and larger structures can be made out with the scanning electron microscope, which is particularly useful for examining fracture surfaces.

When tested in tension, specimens of bone produce a characteristic stress-strain curve: there is a straight elastic region, then a short transitional corner, then a longer, much flatter region, the 'yield region'. The bone is behaving elastically in the first region, as is shown by the fact that if the bone is loaded and unloaded cyclically the stress-strain curves are virtually superimposable. However, if the bone is unloaded and reloaded after the curve has flattened out, the curves are no longer superimposable. Irreversible change has taken place in the bone.

There is evidence that this yield behaviour is caused by tiny cracks forming but being prevented from growing in length very much. The evidence for this comes from a variety of sources: optical effects (Currey & Brear 1974), submicroscopic examination (Carter & Hayes

1977; Ascenzi & Bonucci 1976) and acoustic emission (Yoon *et al.* 1979; Netz *et al.* 1980). As a result of these studies it is reasonable to conclude that the yield behaviour of bone is the result of innumerable cracks which, because of the way the various elements of bone – the secondary osteones, laminae, lamellae, fibrils and so on – are put together, cannot travel very far, and are soon brought to a halt. The ability to crack, yet not fail, accounts for the toughness of bone, such as it is.

The toughness of bone with an intermediate amount of mineral can be explained by the fact that cracks, which tend to form in the highly stressed regions, are always being interrupted as they run through the bone, and do not become long enough to be dangerous. As bone becomes more and more highly mineralized it becomes easier for cracks to travel through it. Although the reason for this increased ease is unknown, it is surely most likely that the mineral is coalescing and tending towards a situation in which it behaves as a single block. As this state is approached the cracks will be interrupted less, because they will not run out of the brittle mineral phase into the more pliant organic phase.

The extremely fibrous appearance of the fracture surface of the two bones with very high work of fracture, the tortoise femur and the deer antler, suggests that these bones may have low modulus fibres of partially mineralized collagen embedded in a high modulus matrix of more highly mineralized collagen (figure 6). On analogy with composites of, for instance, Polypropylene fibres in cement, this arrangement should have a high fracture toughness (Mackley & Parker 1980).

I consider it of the utmost importance that workers on the mineral phase of bone should attempt to characterize the structure of the mineral phase in completely mature bones with various amounts of mineralization. Not until this is done will we be able to understand properly what is going on when bone fractures.

#### COMPARISON OF BONE WITH MOTHER OF PEARL

Although, as we shall see, bone material is well adapted to different kinds of function that it has to perform, when it is highly mineralized it is much less tough than another type of skeletal material with a very high mineral content. This is nacre, or mother of pearl, which is found in the shells of many molluscs. Nacre has about 98% (by mass) calcium carbonate, in the form of aragonite, yet it has a work of fracture of about  $1500 \text{ J m}^{-2}$  (Currey 1979). Nacre achieves this value by completely separating the organic phase from the mineral phase, so that the mineral occurs in sheets about  $0.3 \mu\text{m}$  thick, with the proteinaceous organic matrix sandwiched between the sheets. This arrangement ensures that cracks have to go through an organic phase every  $0.3 \mu\text{m}$ . The energy required for the resultant shearing of the organic phase produces the toughness. Of course, if the crack travels *between* the sheets, the fracture toughness is very low (Currey 1979). Bone is set, as it were, on another path. In bone there is always an intimate relation between the mineral and the organic phases, at the nanometre level. This allows excellent fine tuning of the degree of anisotropy, and the degree of mineralization. However, I think there is a penalty to be paid, that of low fracture toughness at high values of Young modulus.

## FUNCTION OF VARIATION IN THE MECHANICAL PROPERTIES OF BONE

Figure 4 shows how different types of bone have different mechanical properties. I wish to state briefly the functional significance of these differences. There is a great danger of producing plausible *post hoc* explanations for observed differences between the properties of biological materials, but in this case most of the differences in function are clear. (For a splendid, but in my view ill aimed, polemic on this subject see Gould & Lewontin (1979) in the proceedings of a previous Royal Society symposium on the evolution of adaptation by natural selection.)

First consider the femur of the cow, marked b in figure 4. This we can take as a standard bone. The bone needs to be stiff, because that is what long bones are for; to act as stiff levers against the ground. The less stiff they are the less efficient locomotion will be. However, if the material were to become very stiff, by an increase in mineralization, it would become less tough, and more liable to fracture. There will be some optimal value of mineralization, producing sufficient toughness with sufficient stiffness.

The bone of the antler of the red deer is, at its best, a very tough material, though less stiff than the bone of long bones. The antler is used in the rutting season in fights between males for the possession of harems. The critical moment for the antler is when it is smashed against the antler of the opposing male. In the moment of impact, stiffness and static bending strength are unimportant, toughness is all-important. When the males have locked antlers, and are pushing each other around, the antler needs to be reasonably stiff, but more flexibility is allowable than in the femur.

The tympanic bulla of the whale is hidden deep inside the head and is never, in ordinary life, exposed to large forces. Its function is to act as a kind of seismograph mass, to keep still while the rest of the whale vibrates around it. The fact that it is dense and massive helps this, but its stiffness is also important, because this increases the impedance mismatch between the otic bones and the rest of the skull, so preventing sound reaching the inner ear except via the tympanic ligament, the whale's equivalent of the tympanic membrane.

These relations between mechanical properties and function are reasonably clear. Others are less so. If we assume that all bone is optimally designed, then it is difficult to explain the deer's antler bone that has a rather low mineralization, and low strength, stiffness, and fracture toughness. (These are the specimens on the left side of figure 4.) In fact these bones, and the specimen from the crocodile which occupies the same region of the diagram, are somewhat porous, although they are still definitely compact bone. This porosity may be reducing all the mechanical properties to some extent, but is unlikely to be the main part of the explanation. The weak antler specimens come from the ends of the antler, where fracture is unimportant. Nevertheless they are not optimal, and remain an embarrassment.

The bone with consistently highest values for work of fracture was the femur of the Galapagos tortoise *Geochelone elephantopus*. Unfortunately, we know virtually nothing about the mechanical properties of reptile bone, so we do not know whether this tortoise is typical of reptiles. Whether the high work of fracture of the bone of the tortoise, with a correspondingly low value for Young modulus, is adaptive is difficult to say. Tortoises are not fast movers, so locomotory efficiency is not important. These Galapagos tortoises are notoriously clumsy, and for a very long-lived animal the selection pressures on not breaking a bone may be considerable.

The specimens from the King penguin *Aptenodytes patagonica* are remarkable. They have mineralizations at the upper end of the range of cow's bone, and have the greatest bending

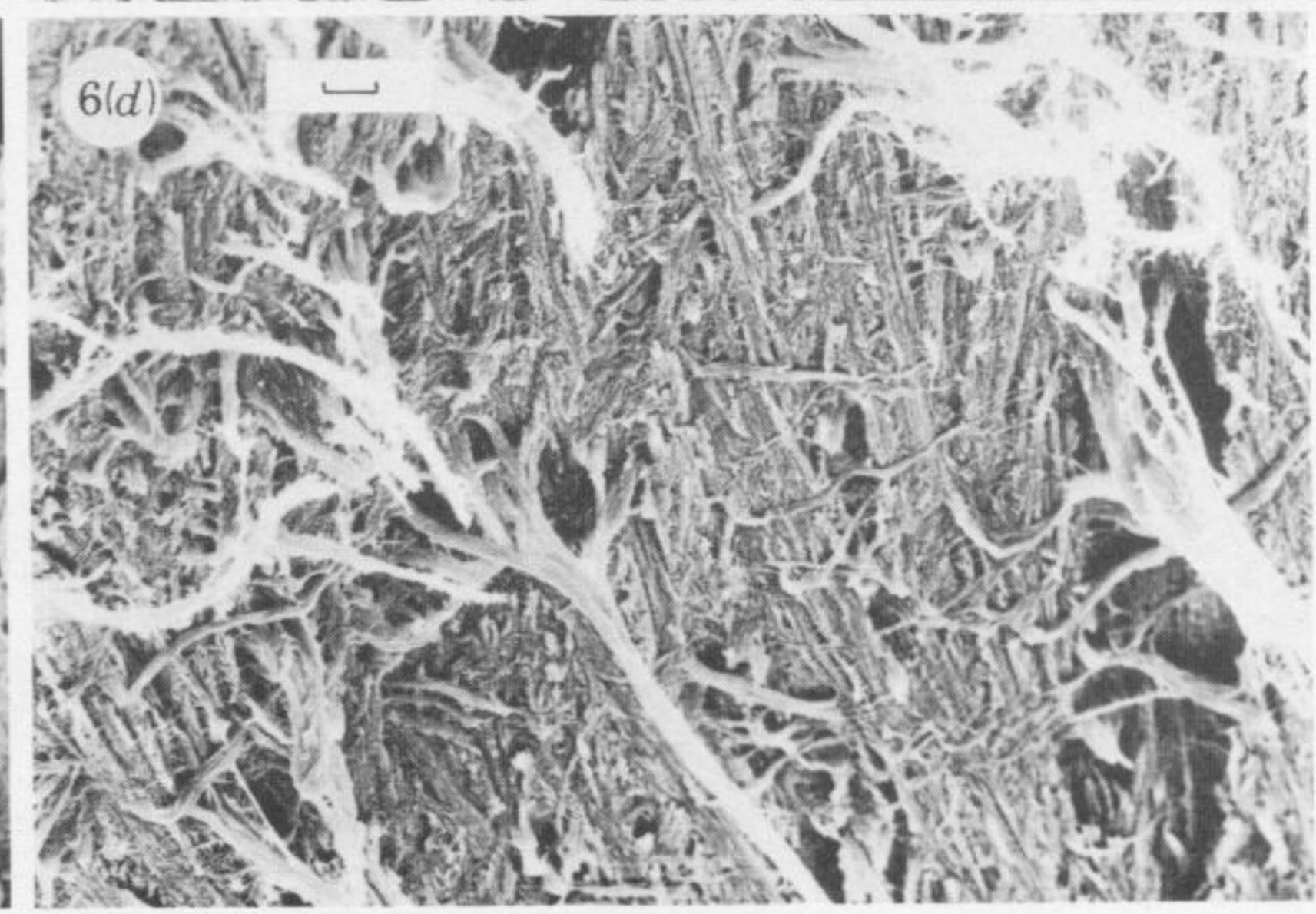
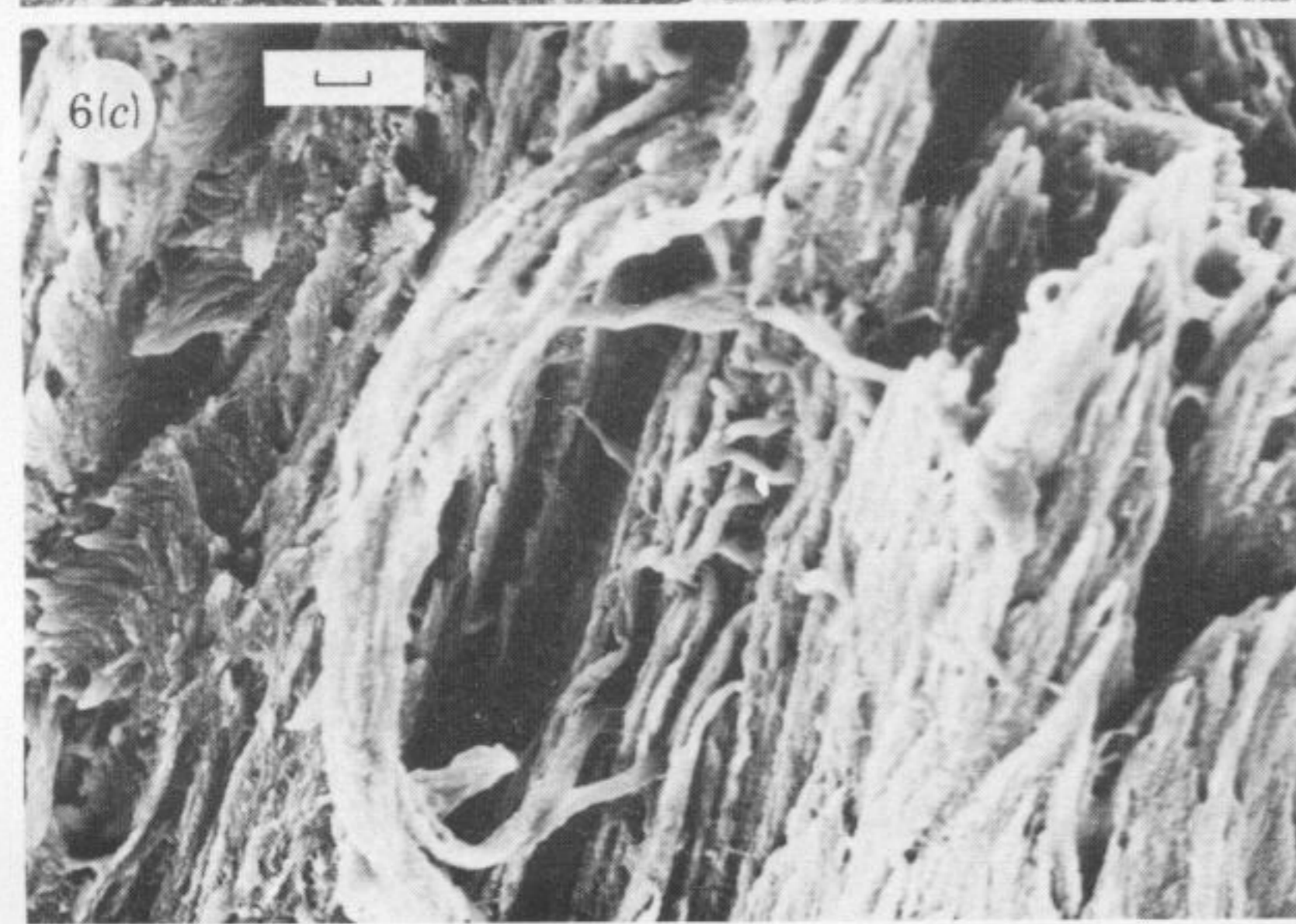
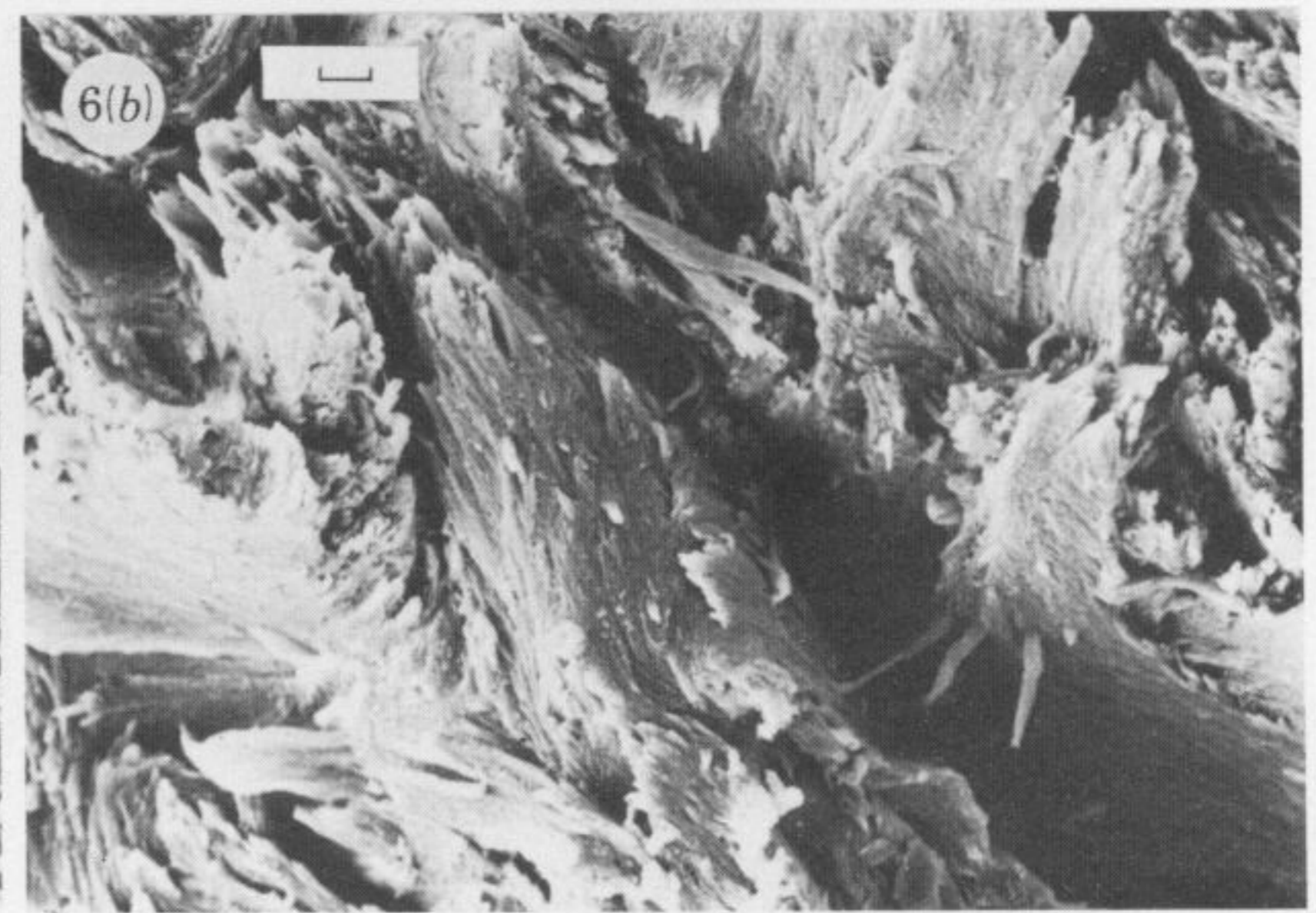
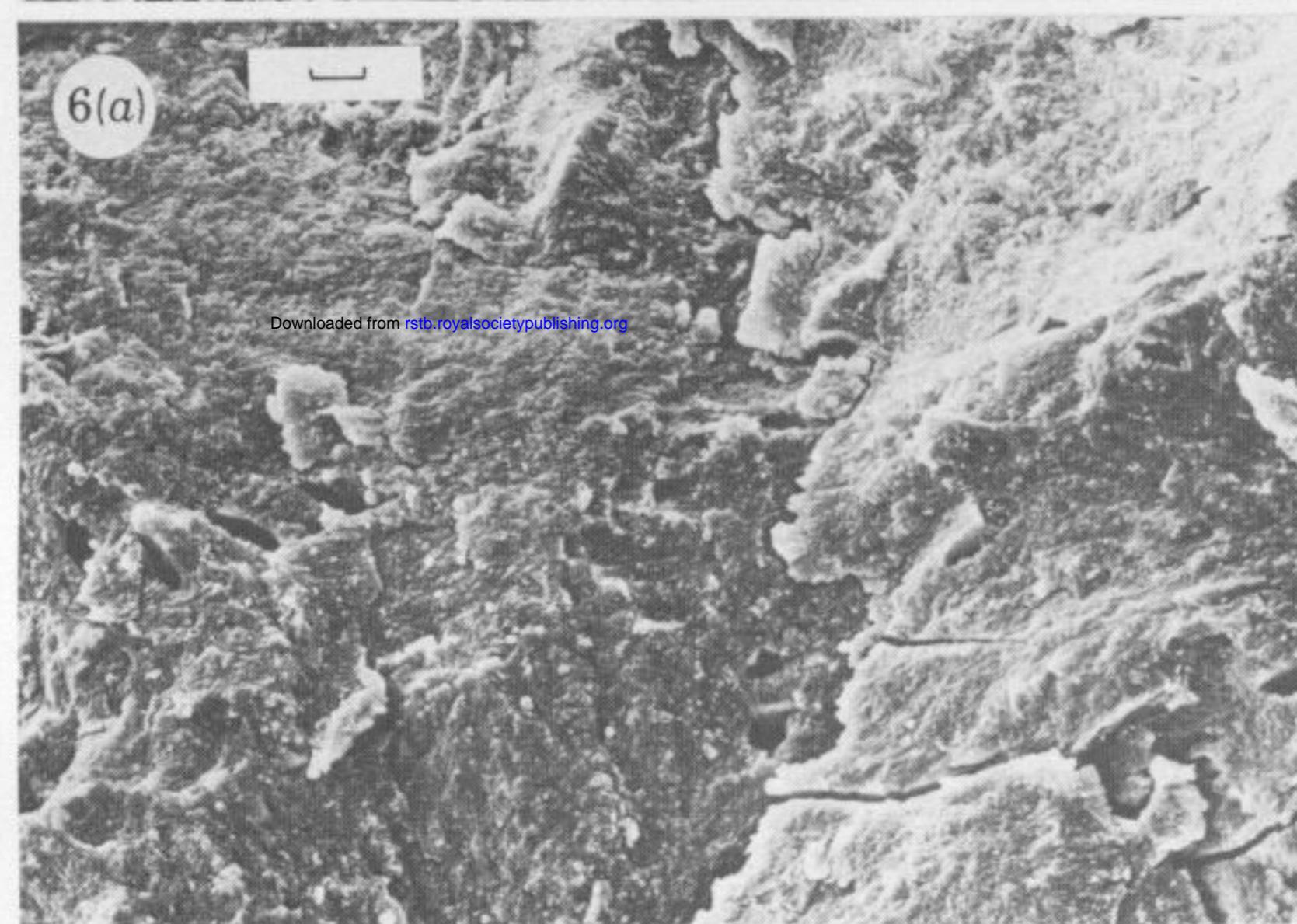
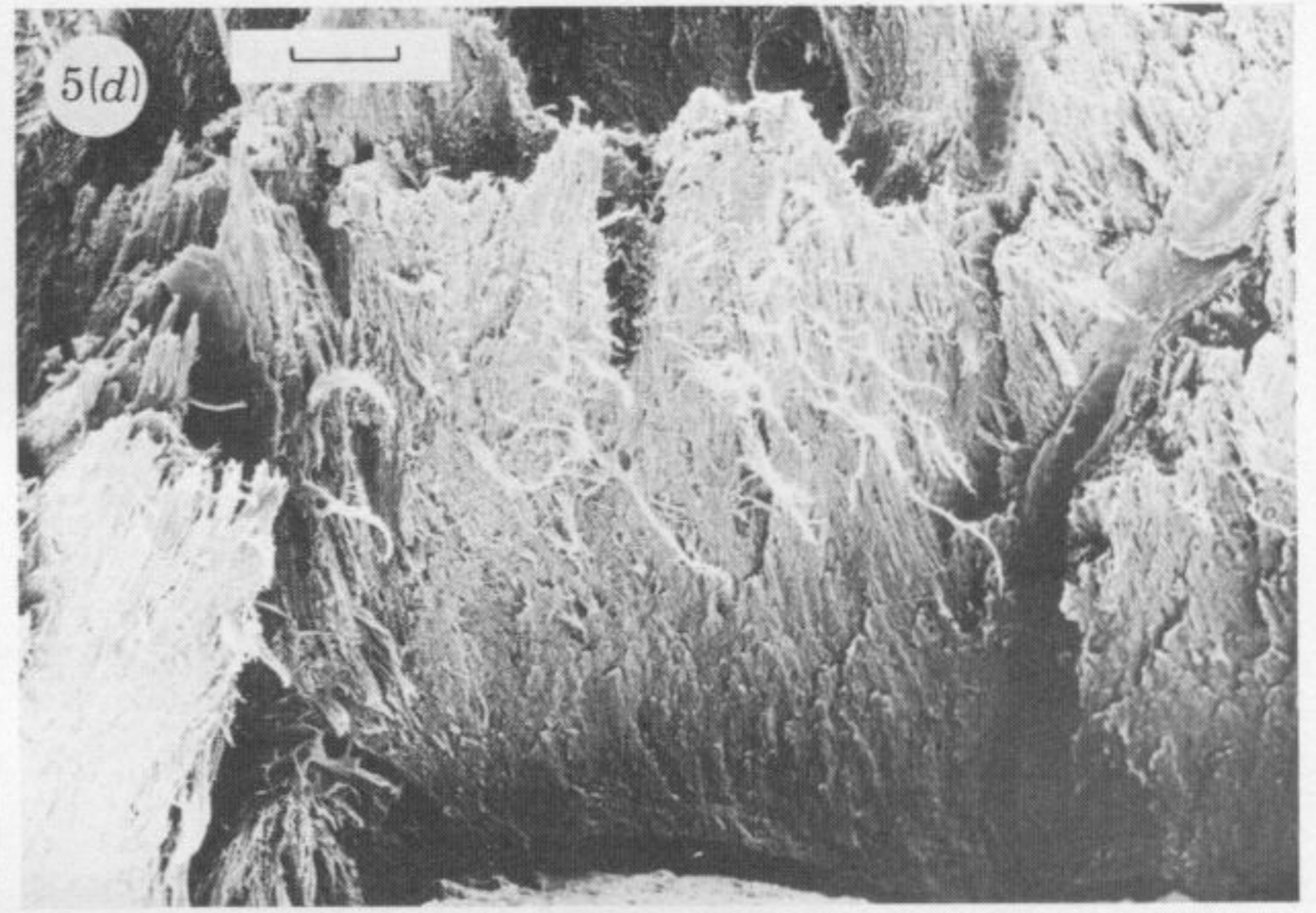
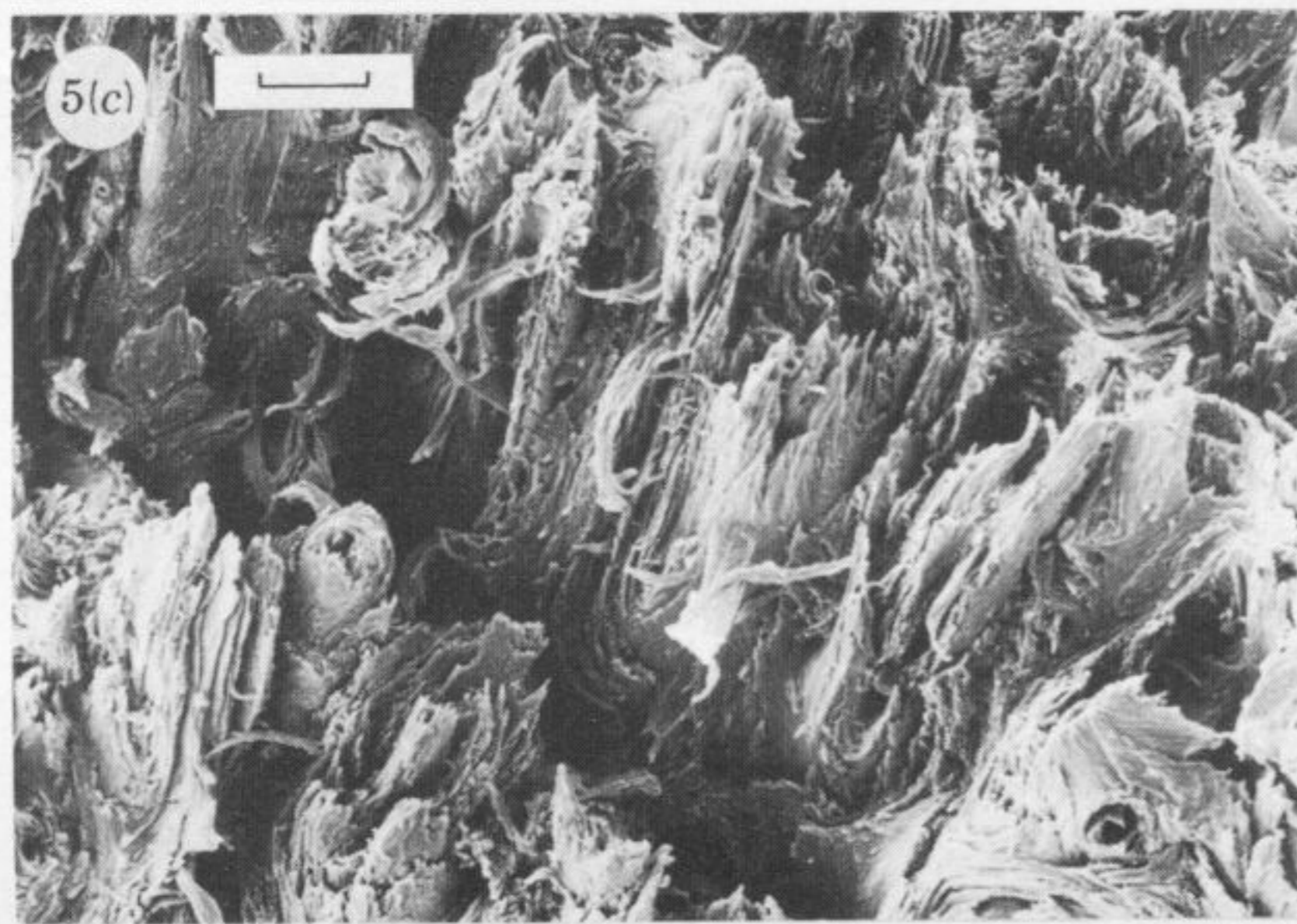
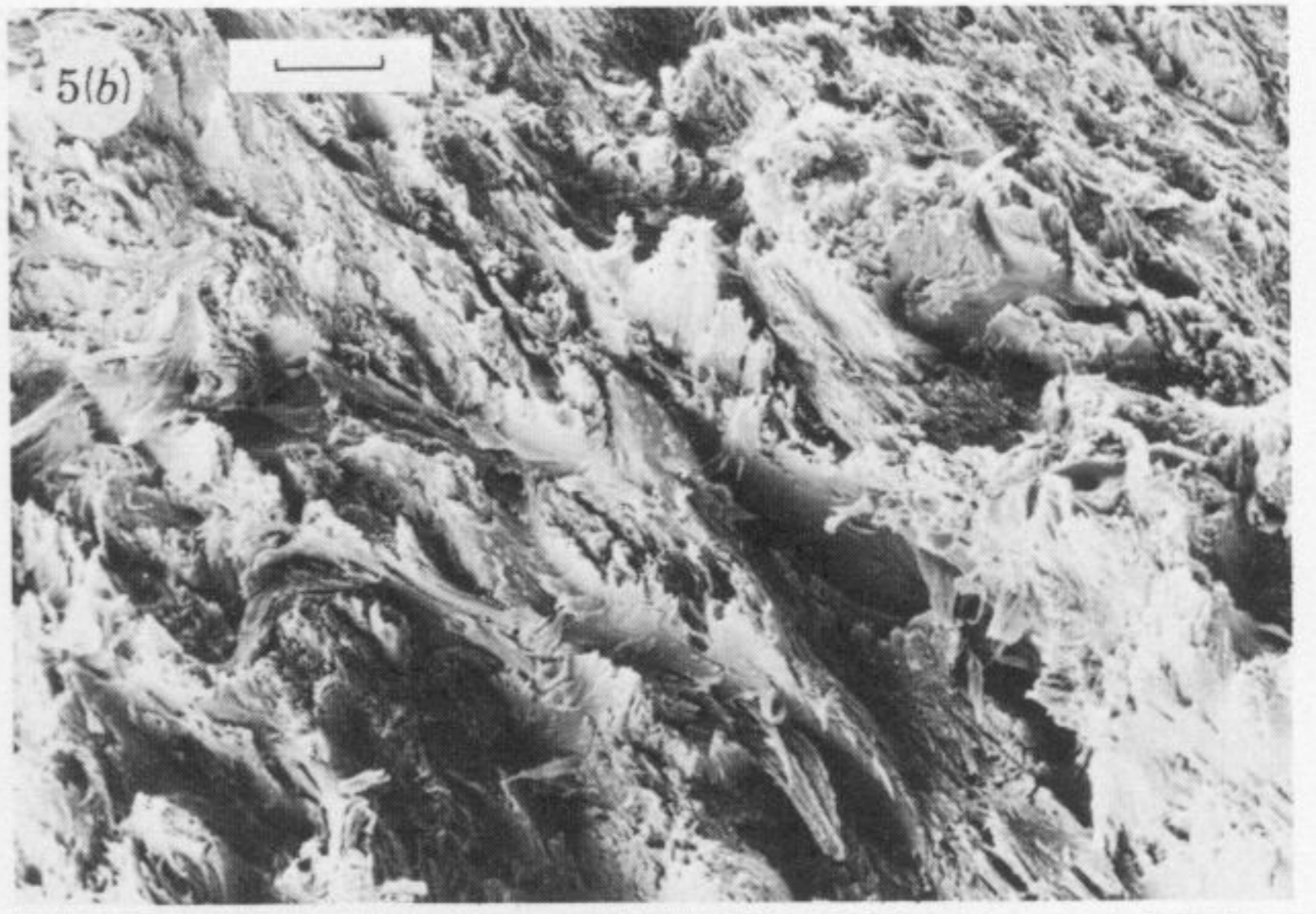
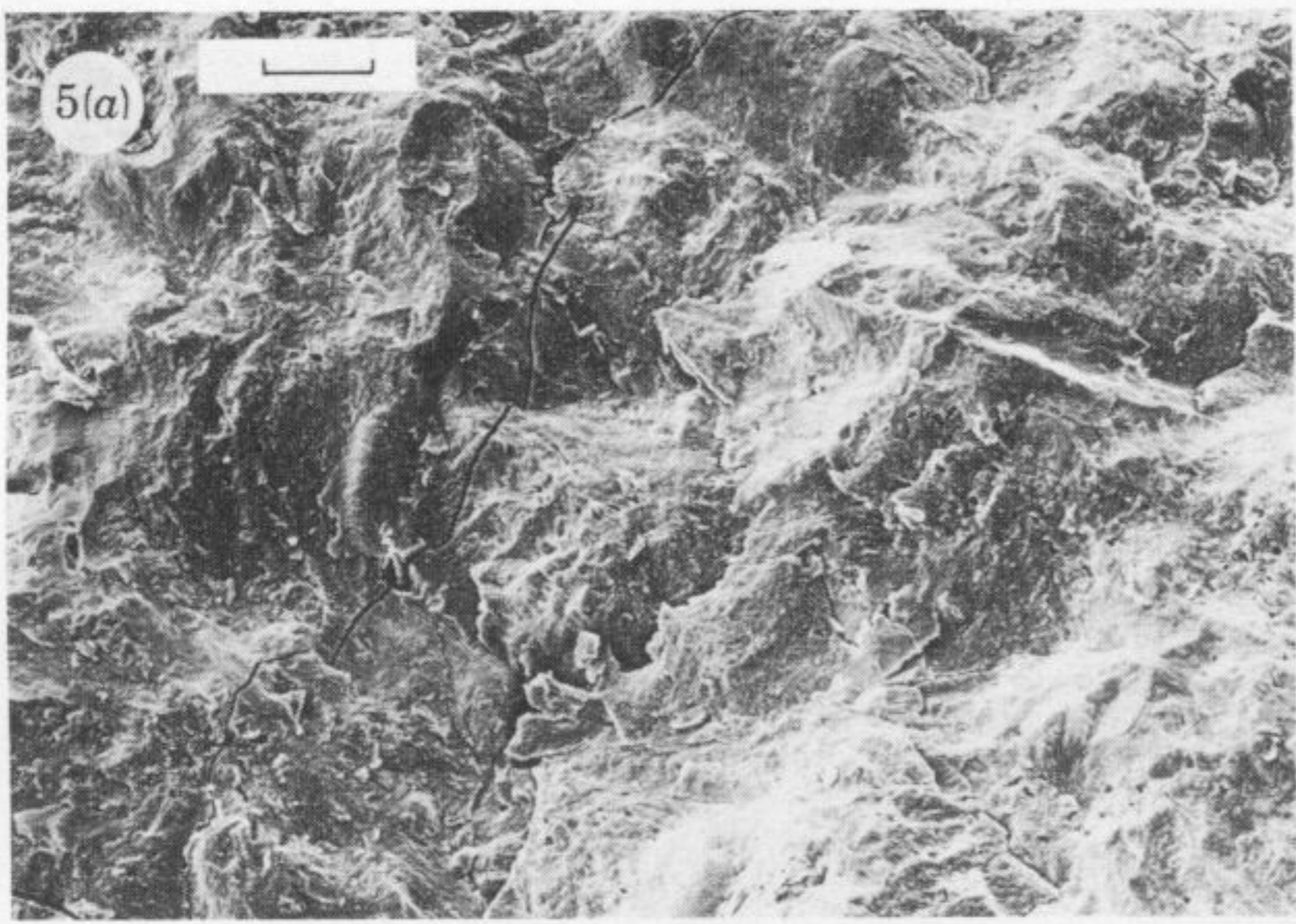
strength and, except for tympanic bulla, the highest Young modulus of any bone I have tested. Yet they are very brittle; it proved difficult to obtain valid work of fracture values, because the specimens tended to break brittly. The explanation may be this. The bones of the arm of a penguin are the skeletal support for the flipper, which is a hydrofoil. The hydrodynamic constraints are such that the hydrofoil must be very slim, and so the bones inside cannot be deep. The forces on the hydrofoil are large, and so the bones need to be stiff. There are two ways of increasing the stiffness of a structure in bending: increase the stiffness of the *material*, and increase the second moment of area of the cross section. But the bones cannot be deep, and they therefore cannot have a large second moment of area. Therefore the stiffness of the structure has to be brought about by it being constructed from a material of high Young modulus.

#### CONCLUSION

This paper has concentrated on the effects of differences of mineralization on the mechanical properties of bone. Other variables are also effective, of course. In particular the anisotropy of bone is determined by its grain – by the arrangement of the collagen fibrils and their associated mineral crystals in space. This has some effect on the Young modulus, and a very marked effect on fracture toughness. Nevertheless, it remains true that the two most important determinants of the mechanical properties of bone are its degree of porosity and its degree of mineralization.

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FIGURES 5 and 6. For description see opposite.